Appendix A:

Equation of Motion

$$Power = Velocity * [Drag + Inertial + Rolling]_{Force} + \frac{(Gravity)_{work} + (Misc)_{work}}{\Delta time}$$

Aerodynamic term:

$$V\left\{\left[\frac{1}{2} r V^2 C_d A_w\right]_{FrontWheel} + (1 - protection)\left[\frac{1}{2} r V^2 C_d A_w\right]_{\mathrm{Re}\, arWheel} + \left[\frac{1}{2} r V^2 C_d A_w\right]_{Frame} + \left[\frac{1}{2} r V^2 C_d A_w\right]_{rider}\right\}$$

Where:

r = air density
V = velocity
C_d = drag coefficient
A = reference area (side area for wheels, frontal area for rider and frame)
Protection = reduction in C_d of rear wheel due to its proximity to frame = .25

Inertial term:

$$w[Ia]_{Frontwheel} + w[Ia]_{\mathrm{Re}\, arwheel} + V \cdot M_{Tot} a$$

Converting to linear acceleration terms and discretizing we have:

$$w = \frac{V}{R}$$

$$a = \frac{dw}{dt} = \frac{d}{dt}\left(\frac{V}{R}\right) = \frac{1}{R}\frac{\Delta V}{\Delta t}$$

$$a = \frac{\Delta V}{\Delta t}$$

$$\therefore$$

$$V\left\{\left[I\frac{1}{R^2}\frac{\Delta V}{\Delta t}\right]_{Frontwheel} + \left[I\frac{1}{R^2}\frac{\Delta V}{\Delta t}\right]_{\mathrm{Re}\, arwheel} + M_{Tot}\frac{\Delta V}{\Delta t}\right\}$$

Where:
w = rotational velocity (rad/s)
a = angular acceleration
R = wheel radius
Dt = change in time
DV = change in velocity
M_{Tot} *= total mass including bike, rider, wheels, tires*
I = moment of inertia

Rolling resistance term:

$$V\left[C_{rr} M_{Tot} g\right]$$

Where:
C_{rr}*= coefficient of rolling resistance*
g = gravitational acceleration (9.81 m/s^2*)*

Gravitational term:

$$M_{Tot} g \frac{\Delta h}{\Delta t}$$

Where:
Dh = change in elevation

Final equation of motion:

$$V\left\{\begin{array}{l}\left[\frac{1}{2} r V^2 C_d A_w\right]_{FrontWheel} + (1 - protection)\left[\frac{1}{2} r V^2 C_d A_w\right]_{\mathrm{Re}\, arWheel} + \left[\frac{1}{2} r V^2 C_d A_w\right]_{Frame} + \left[\frac{1}{2} r V^2 C_d A_w\right]_{rider} \\ + \left[I \frac{1}{R^2} \frac{\Delta V}{\Delta t}\right]_{Frontwheel} + \left[I \frac{1}{R^2} \frac{\Delta V}{\Delta t}\right]_{\mathrm{Re}\, arwheel} + M_{Tot} \frac{\Delta V}{\Delta t} \\ + C_{rr} M_{Tot} g \\ \end{array}\right\}$$

$$+ M_{Tot} g \frac{\Delta h}{\Delta t} = Power_{rider}$$